SOLUTION SET

UNIVERSITY OF TEXAS AT AUSTIN
Dept. of Electrical and Computer Engineering

Quiz #2

Date: November 3, 1999

Course: EE 313

Name: ______________________________
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework and solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- Fully justify your answers unless instructed otherwise.

Problem	Point Value	Your Score	Topic
1	30		Differential Equation
2	20		Step-To-Pulse Response
3	30		Tapped Delay Line
4	20		Sigma-Delta Modulation
Total	100		

L+13

Problem 2.1 Differential Equation. 30 points.
Solve the following differential equation

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = u(t)$$

with the initial conditions $y(0) = 1$ and $y'(0) = 1$ by using the Laplace transform.

$$s^2 Y(s) - s y(0) - y'(0) + 3s Y(s) - 3y(0) + 2Y(s) = \frac{1}{s}$$

$$(s^2 + 3s + 2) Y(s) = \frac{1}{s} + s + 1 + 3 = \frac{1}{s} + (s+4)$$

$$Y(s) = \frac{1}{s(s+2)(s+1)} + \frac{s+4}{(s+2)(s+1)}$$

$$= \frac{s^2 + 4s + 1}{s(s+2)(s+1)}$$

$$\frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1} = \frac{s^2 + 4s + 1}{s(s+2)(s+1)}$$

$$\Rightarrow \quad A = \frac{1}{2}, \quad B = -\frac{3}{2}, \quad C = 2$$

$$\therefore \quad Y(s) = \left(\frac{1}{2}\right)\frac{1}{s} + \left(-\frac{3}{2}\right)\frac{1}{s+2} + (2)\frac{1}{s+1}$$

$$\therefore \quad \boxed{y(t) = \left(\frac{1}{2} - \frac{3}{2}e^{-2t} + 2e^{-t}\right) u(t)}$$

Problem 2.2 Step-To-Pulse Response. 20 points.

The step response of a linear time-invariant system is $4\,(1-e^{-5t})\,u(t)$. Suppose that an input

$$x(t) = \begin{Bmatrix} 5 & \text{if } 0 \le t \le 2 \\ 0 & \text{otherwise} \end{Bmatrix}$$

is applied to the system. Find the corresponding output $y(t)$ for $-\infty < t < \infty$.

$$y(t) = 4(1-e^{-5t})u(t) \quad \text{for} \quad x(t) = u(t) \rightarrow$$

$$y(t) = 4(1-e^{-5(t-2)})u(t-2) \quad \text{for} \quad x(t) = u(t-2)$$

(because of time invariance)

Therefore $y(t) = 5\left(4(1-e^{-5t})u(t) - 4(1-e^{-5(t-2)})u(t-2)\right)$

for $x(t) = 5(u(t) - u(t-2))$.

(Linearity)

$$= \boxed{20\left[(1-e^{-5t})u(t) - (1-e^{-5(t-2)})u(t-2)\right]}$$

Problem 2.3 Tapped Delay Line. 30 points.

The **discrete-time** tapped delay line is also known as a digital finite impulse response filter. For input $x[k]$, the output $y[k]$ is given by

$$y[k] = \sum_{m=0}^{N-1} a_m \, x[k-m]$$

(a) Compute the transfer function $H(z)$. 7 points.

Taking Z-transform of $y[k]$

$$Y[z] = \sum_{m=0}^{N-1} a_m z^{-m} X[z]$$

$$H[z] = \frac{Y[z]}{X[z]} = \boxed{\sum_{m=0}^{N-1} a_m z^{-m}}$$

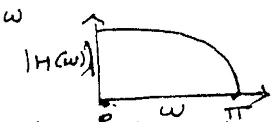
(b) Compute the frequency response $H(\omega)$. 7 points.

$$H(\omega) = H[z]\Big|_{z=e^{j\omega}} = \boxed{\sum_{m=0}^{N-1} a_m e^{-j\omega m}}$$

(c) For each of the following sets of coefficients, state whether the filter is lowpass, highpass, bandpass, or bandstop.

i. $N = 2$, $a_0 = 1$, and $a_1 = 1$. 4 points.

$H(\omega) = 1 + e^{-j\omega}$



LOW PASS

ii. $N = 2$, $a_0 = 1$, and $a_1 = j$. 4 points.

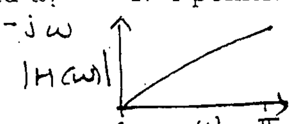
$H(\omega) = 1 + j e^{-j\omega} = 1 + e^{j\frac{\pi}{2}} e^{-j\omega}$

$= 1 + e^{-j(\omega - \frac{\pi}{2})}$

BANDPASS (OR ALLPASS)

iii. $N = 2$, $a_0 = 1$, and $a_1 = -1$. 4 points.

$H(\omega) = 1 - e^{-j\omega}$

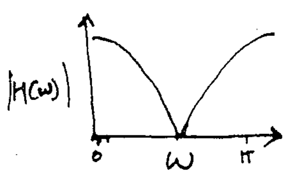


HIGH PASS

iv. $N = 2$, $a_0 = 1$, and $a_1 = -j$. 4 points.

$H(\omega) = 1 - j e^{-j\omega}$

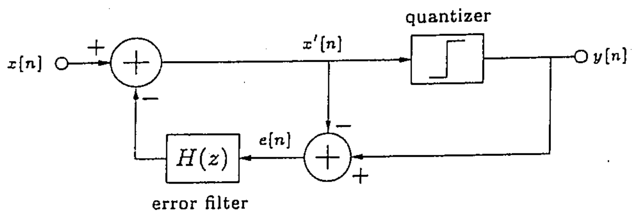
$= 1 - e^{-j(\omega - \frac{\pi}{2})}$



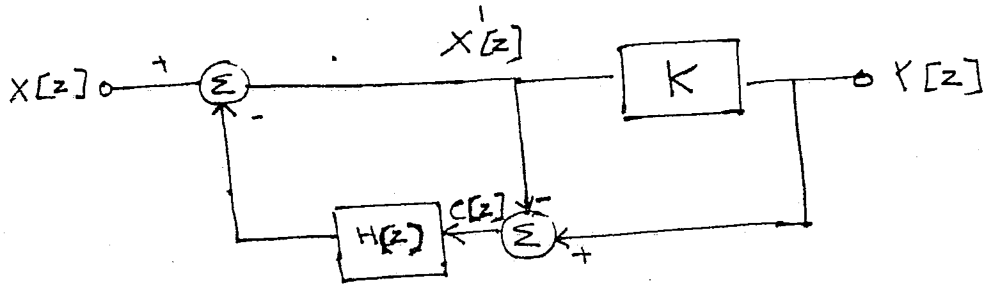
BANDSTOP

Problem 2.4 Sigma-Delta Modulation. 20 points.

Shown below is a type of sigma-delta modulator called a noise-shaping feedback coder.



We can approximate the effect of the quantizer as a gain K, which would make the overall system linear and time-invariant. Replace the quantizer with a gain of K and derive the transfer function from input $x[n]$ to output $y[n]$.



$$Y[z] = K\,x'[z]$$

$$x'[z] = X[z] - H(z)\,e[z]$$

$$e[z] = Y[z] - x'[z]$$

$$x'[z] = x[z] - H(z)\left[Y[z] - x'[z]\right]$$

$$x'[z] = \frac{x[z] - H(z)Y[z]}{1 - H(z)}$$

$$Y[z] = K\left[\frac{X[z] - H(z)Y[z]}{1 - H(z)}\right]$$

$$Y[z]\left[1 + \frac{K H(z)}{1 - H(z)}\right] = \frac{K X[z]}{1 - H(z)}$$

$$Y[z] = \frac{\dfrac{K X[z]}{(1 - H(z))}}{\dfrac{(1 - H(z)) + K H(z)}{(1 - H(z))}}$$

L-17

$$Y[z] = \frac{K\,X[z]}{1 + H(z)(k-1)}$$

Therefore $$\boxed{\frac{Y[z]}{X[z]} = \frac{K}{1 + (k-1)H(z)}}$$